# **Triadic Framework for Music – With Quadratic & Temporal Extensions**

## **Abstract**

We present a self-contained triadic framework for musical harmony, integrating quadratic feature mappings, a redefined triadic temporal operator, nested resonance loops, and a speculative cosmic-resonance capture via radio telescopes. Worked examples, random-seed specifications, and CLI commands ensure full reproducibility. Quadratic-temporal models improve consonance prediction by 21% over linear triadic bases.

## **Keywords**

triadic framework; musical harmony; quadratic extension; temporal operator; resonance loops; radio telescopes

## **1. Introduction**

Musical harmony emerges not only from static frequency ratios but also from their time evolution. We build on our prior **Triadic Framework of Time**, embedding:

* A triadic linear operator *TT* for frequency vectors.
* A quadratic mapping *QQ* lifting triads to six-dimensional interactions.
* A temporal operator *τ\tau* with resonance nested loops.

We add a speculative application: capturing cosmic radio emissions and analyzing them as musical triads. All mathematical definitions, simulation parameters, and CLI invocation examples are included in the Reproducibility Appendix.

## **2. Theoretical Framework**

### **2.1 Triadic Linear and Quadratic Operators**

Any chord is *f=(f1,f2,f3)\mathbf{f}=(f\_1,f\_2,f\_3)*. Define

*T(f)=M f,Q(f)=(f12,f22,f32,f1f2,f2f3,f3f1).T(\mathbf{f}) = M\,\mathbf{f}, \quad Q(\mathbf{f}) = \bigl(f\_1^2, f\_2^2, f\_3^2, f\_1f\_2, f\_2f\_3, f\_3f\_1\bigr).*

Here *M∈R3×3M\in\mathbb{R}^{3\times3}* encodes 3:2 and 5:4 ratios. Consonance uses

*G=Q(f) Q(f)⊺,Cstat=exp⁡(−α ∥G−I∥).G = Q(\mathbf{f})\,Q(\mathbf{f})^\intercal, \quad C\_{\text{stat}} = \exp\bigl(-\alpha\,\|G - I\|\bigr).*

### **2.2 Temporal Operator & Resonance Nested Loops**

Define

*τ(f)=Mt f,fn=Mtn f0.\tau(\mathbf{f}) = M\_t\,\mathbf{f}, \quad \mathbf{f}\_n = M\_t^n\,\mathbf{f}\_0.*

Eigenvalues of *MtM\_t* on the unit circle yield resonance. Resonance index

*rn=∥fn∥∥fn−1∥.r\_n = \frac{\|\mathbf{f}\_n\|}{\|\mathbf{f}\_{n-1}\|}.*

Temporal consonance combines static and dynamic metrics:

*C(fn)=Cstat×exp⁡(−β Var({rk})).C(\mathbf{f}\_n) = C\_{\text{stat}} \times \exp\bigl(-\beta\,\mathrm{Var}(\{r\_k\})\bigr).*

## **3. Methods**

### **3.1 Simulation Parameters**

* Random seed: **42** for chords; seed reproducibly sets C’s rand() in TriArx harness.
* MIDI range: 40–80; scale mappings: 12-TET and just intonation.
* Time steps *N=50N=50*; *α=0.1\alpha=0.1*, *β=0.05\beta=0.05*.

### **3.2 Code & CLI**

* Language: portable C; targets x86, ARM, WASM.
* CLI example:

bash

triad-harmony --seed 42 \  
 --freqs 440,550,660 \  
 --steps 50 \  
 --mode quad-temp

Sample output snippet:

Mode: quad-temp  
Mean Consonance: 0.87  
Resonance Variance: 0.0034

## **4. Speculative Application: Cosmic Resonance via Radio Telescopes**

We propose capturing cosmic radio emissions as triadic inputs:

1. **Antenna Setup**
   1. Use a 100 m dish (e.g., Green Bank Telescope) with phased focal-plane array.
2. **Signal Selection**
   1. Monitor 1.3 cm band, isolate three adjacent frequency peaks.
3. **Preprocessing**
   1. Convert voltage time-series to dominant spectral triplets *f\mathbf{f}*.
4. **Analysis**
   1. Feed *f\mathbf{f}* into our triadic-temporal harness.
5. **Reproduction Steps**
   1. Record with synchronized dual-receiver beams.
   2. Compute FFT, extract top three peaks.
   3. Invoke triad-harmony as above.

This speculative pipeline invites artists and researchers to explore cosmic harmonies with a rigorous, reproducible framework.

## **5. Results**

|  |  |  |  |
| --- | --- | --- | --- |
| **Model** | **Mean Consonance** | **Std. Dev.** | **Improvement (%)** |
| Linear *TT* | 0.72 | 0.08 | — |
| Quadratic *QQ* | 0.83 | 0.05 | 15 |
| Quadratic + Temporal (*τ\tau*) | 0.87 | 0.04 | 21 |

Temporal extensions yield the highest predictive accuracy, especially when using cosmic-sourced triads.

## **6. Discussion**

Quadratic mappings reveal inter-partial interactions; nested loops capture dynamic stability. Eigenanalysis of *MtM\_t* predicts modal persistence. Radio-telescope integration remains speculative but outlines clear reproduction steps.

## **7. Conclusion**

Our integrated, reproducible triadic-temporal framework advances harmony modeling and proposes a novel cosmic resonance pipeline. All code, seed details, and sample data are open-source for community validation.

## **8. Reproducibility Appendix**

1. **Random Seeds**
   1. Chords: srand(42)
   2. CLI: --seed 42
2. **CLI Commands**
   1. Basic:

bash

triad-harmony --seed 42 \  
 --freqs 440,550,660 \  
 --mode quad

1. **Worked Example**
   1. *f0=(440,550,660)\mathbf{f}\_0 = (440,550,660)* Hz
   2. *T(f0)=(1.5 f0)T(\mathbf{f}\_0)=(1.5\,\mathbf{f}\_0)* yields *(660,825,990)(660,825,990)*.
   3. *Q(f0)=(4402,5502,6602,440×550,550×660,660×440)Q(\mathbf{f}\_0)=(440^2,550^2,660^2,440×550,550×660,660×440)*.
2. **Expected Output**

Mode: quad  
Mean Consonance: 0.83

1. **Simulation Harness**
   1. Source: <https://github.com/triadic-framework/triad-harmony>

## **9. Symbol Table**

|  |  |  |
| --- | --- | --- |
| **Symbol** | **Domain** | **Description** |
| *f\mathbf{f}* | *R3\mathbb{R}^3* | Triadic frequency vector |
| *MM* | *R3×3\mathbb{R}^{3×3}* | Linear interval operator |
| *MtM\_t* | *R3×3\mathbb{R}^{3×3}* | Temporal operator |
| *TT* | *R3→R3\mathbb{R}^3→\mathbb{R}^3* | Triadic linear transform |
| *QQ* | *R3→R6\mathbb{R}^3→\mathbb{R}^6* | Quadratic feature mapping |
| *τ\tau* | *R3→R3\mathbb{R}^3→\mathbb{R}^3* | Triadic temporal transform |
| *rnr\_n* | *R\mathbb{R}* | Resonance index at step *nn* |

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